B.Tech. Degree V Semester Examination November 2012

IT/CS/EC/CE/ME/SE/EB/EI/EE/FT 501 ENGINEERING MATHEMATICS IV
(2006 Scheme)

Time : 3 Hours   Maximum Marks : 100

PART A
(Answer *ALL* questions)

(8 × 5 = 40)

I. (a) A coin is tossed 10 times . Let X be the number of heads occurring. Use binomial distribution to find the mean and variance of X.

(b) If X is a random variable following Poisson distribution and $3P[x=2]=2P[x=1]$, find $P[x=0]$ and the standard deviation of X.

(c) A random sample of size 12 is taken from a normal population with variance 9. Find the probability that the sample variance lies between 3.4 and 14.8.

(d) A random sample of size 36 is taken from a normal population with standard deviation 3. Find the probability that the sample mean exceeds the population mean by atleast one.

(e) Prove that $\mu^2 = 1+\frac{\delta^2}{4}$.

(f) Compute $\int_1^2 \frac{dx}{x}$ using trapezoidal rule by taking h=0.25.

(g) Solve the initial value problem $\frac{dy}{dx} = 1+xy;\ y(0)=1$ at $x=0.1$ correct to four decimal places by Taylor series method.

(h) Give $\frac{dy}{dx} = x^3+y;\ y(0)=1$. Compute $y(0.02)$ using Euler's method by taking $h=0.01$.

PART B

(4 × 15 = 60)

II. (a) If $f(x)=\frac{k}{1+x^2},\ -\infty<x<\infty$ is the probability density function of a random variable X, find the value of k and evaluate $P\left[1<X<\sqrt{3}\right]$. (8)

(b) A random variable X is uniformly distributed in $(-\alpha,\alpha),\ \alpha>0$. Find the value of α if $P[|X|<1]=P[|X|>1]$. (7)

OR

III. (a) Determine the coefficient of correlation between x and y and the means of x and y if the regression lines for x and y are $3x+2y=26;\ 6x+y=31$. (8)

(b) Use the method of least squares to fit a straight line to the following data. (7)

x	1	2	3	4	5
y	14	13	9	5	2

IV. (a) A sample of size 100 is taken from a population with $\sigma=5.1$. The sample mean $\bar{x}=21.6$. Construct a 95% confidence interval for the population mean. (7)

(b) A company claims that its light bulbs are superior to those of its main competitor. A study showed that a sample of $n_1=40$ of its bulbs has a mean lifetime of 647 hours of continuous use with a standard deviation of 27 hours, while a sample of size $n_2=40$ bulbs made by its main competitor has a mean lifetime of 638 hours of continuous use with a standard deviation of 31 hours. Can we accept the claim of the company at 0.05 level of significance? (8)

OR

V. (a) A population follows normal distribution with mean μ and variance 9. To test $H_0 : \mu = 5$ against $H_1 : \mu = 7$, the test procedure suggested is to reject H_0 if $\bar{x} \geq 6$ where $\bar{x}$ is the mean of a sample of size 16. Find the significance level and the power of the test. (8)

(b) The standard deviations of two samples of sizes 10 and 14 from two normal populations are 3.5 and 3 respectively. Test whether the standard deviations of the populations are equal at 0.02 level of significance. (7)

VI. (a) From the following table of half yearly premium for policies maturing at different ages, estimate the premium for policies maturing at ages 46 and 63. (8)

Age (x)	45	50	55	60	65
Premium (y)	1150	960	830	740	680

(b) Compute $\frac{dy}{dx}$ at $x = 1$ and $x = 6$ from the following data. (7)

x	1	2	3	4	5	6
y	1	8	27	64	125	216

OR

VII. (a) Evaluate $\int_0^6 \frac{dx}{1+x}$ using (8)

(i) Simpson's $\frac{1}{3}$ rule (ii) Simpson's $\frac{3}{8}$ rule.

(b) Use Newton's divided difference formula to find $f(9)$ from the following data. (7)

x	5	7	11	13	17
$f(x)$	150	392	1452	2366	5202

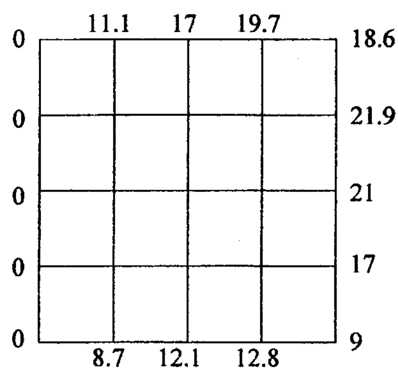
VIII. (a) Solve by modified Euler's method $\frac{dy}{dx} = x^2 + y$; $y(0) = 1$ at $x = .02$ by taking $h = .01$. (8)

(b) Using Bender Schmidt method, solve $\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial u}{\partial t} = 0$ when $u(0,t) = 0 = u(4,t)$ and $u(x,0) = x(4-x)$. Assuming $h = 1$, find the values of u up to $t = 5$. (7)

OR

IX. (a) Apply Runge Kutta method to find an approximate value of y when $x = 0.2$ in steps of 0.1 if $\frac{dy}{dx} = x + y$; $y(0) = 1$. (5)

(b) Solve the elliptic equation $U_{xx} + U_{yy} = 0$ for the following square mesh with boundary values as shown below: (10)



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